b) Let $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be the Linear Transformation with

$$[T]_{\mathcal{X}\mathcal{X}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

compute: $[T]_{y\mathcal{X}}$, $[T]_{\mathcal{X}y}$ and $[T]_{yy}$

For $\mathcal{X}\mathcal{X}$: $T(1,0,0) = 1(1,0,0) + 0(0,1,0) + 0(0,0,1) = (1,0,0)$

$T(0,1,0) = 0(1,0,0) + 2(0,1,0) + 0(0,0,1) = (0,2,0)$

$T(0,0,1) = 0(1,0,0) + 0(0,1,0) + 3(0,0,1) = (0,0,3)$

Now:

- $[T]_{y\mathcal{X}}$: $T(1,0,0) = (1,0,0) = 0(1,1,0) + -1(0,0,1) + 1(1,0,1) = (1,0,0)$

  $T(0,1,0) = (0,2,0) = 2(1,1,0) + 2(0,0,1) + -2(1,0,1) = (0,2,0)$

  $T(0,0,1) = (0,0,3) = 0(1,1,0) + 3(0,0,1) + 0(1,0,1) = (0,0,3)$

$$[T]_{y\mathcal{X}} = \begin{pmatrix} 0 & 2 & 0 \\ -1 & 2 & 3 \\ 1 & -2 & 0 \end{pmatrix}$$

- $[T]_{\mathcal{X}y}$: $T(1,1,0) = T(1,0,0) + T(0,1,0) + 0T(0,0,1) = (1,2,0)$

  $T(0,0,1) = 0(1,0,0) + 0(0,1,0) + T(0,0,1) = (0,0,3)$

  $T(1,0,1) = T(1,0,0) + 0(0,1,0) + T(0,0,1) = (1,0,3)$

$$[T]_{\mathcal{X}y} = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 0 & 0 \\ 0 & 3 & 3 \end{pmatrix}$$

- $[T]_{yy}$ = $T(1,1,0) = (1,2,0) = 2(1,1,0) + 1(0,0,1) + -1(1,0,1) = (1,2,0)$

  $T(0,0,1) = (0,0,3) = 0(1,1,0) + 3(0,0,1) + 0(1,0,1) = (0,0,3)$

  $T(1,0,1) = (1,0,3) = 0(1,1,0) + 2(0,0,1) + 1(1,0,1) = (1,0,3)$

$$[T]_{yy} = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 3 & 2 \\ -1 & 0 & 1 \end{pmatrix}$$