Workshop 7

1a. $T(1,0,0) = 0(1,1,0) - 1(0,0,1) + 1(1,0,1)$

$T(0,1,0) = 1(1,1,0) + 1(0,0,1) - 1(1,0,1)$

$T(0,0,1) = 0(1,1,0) + 1(0,0,1) + 0(1,0,1)$

$$[T]_{yx} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 1 & 1 \\ 1 & -1 & 0 \end{bmatrix} = [id_{\mathbb{R}^3}]_{yx}$$

$$T(1,1,0) = 1(1,0,0) + 1(0,1,0) + 0(0,0,1)$$
$$T(0,0,1) = 0(1,0,0) + 0(0,1,0) + 1(0,0,1)$$
$$T(1,0,1) = 1(1,0,0) + 0(0,1,0) + 1(0,0,1)$$

$$[T]_{xy} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} = [id_{\mathbb{R}^3}]_{xy}$$

$$[id_{\mathbb{R}^3}]_{yx}[id_{\mathbb{R}^3}]_{xy} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$$[id_{\mathbb{R}^3}]_{xy}[id_{\mathbb{R}^3}]_{yx} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

The change of basis matrices $[id_{\mathbb{R}^3}]_{xy}$ and $[id_{\mathbb{R}^3}]_{yx}$ are inverses of each other.

b. $x \longrightarrow y$

$$[T]_{yx} = [id_{\mathbb{R}^3}]_{yx}[T]_{xx}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ -1 & 1 & 1 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$[T]_{yx} = \begin{bmatrix} 0 & 2 & 0 \\ -1 & 2 & 3 \\ 1 & -2 & 0 \end{bmatrix}$$

$y \to x$

$$[T]_{xy} = [T]_{xx}[id_{\mathbb{R}^3}]_{xy}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$[T]_{xy} = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 0 & 0 \\ 0 & 3 & 3 \end{bmatrix}$$

$y \to y$

$$[T]_{yy} = [id_{\mathbb{R}^3}]_{yx}[T]_{xx}[id_{\mathbb{R}^3}]_{xy}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ -1 & 1 & 1 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 & 0 \\ -1 & 2 & 3 \\ 1 & -2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$[T]_{yy} = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 3 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$