Instructions

1. The statements in Italics are for introducing results and notations that may be used again in this course. You are only required to read and think about them.

2. To receive full credit you must explain how you got your answer.

3. While I encourage collaboration, you must write solutions IN YOUR OWN WORDS. DO NOT SHARE COMPLETE SOLUTIONS before they are due. YOU WILL RECEIVE NO CREDIT if you are found to have copied from whatever source or let others copy your solutions.

4. Workshops must be handwritten (electronic handwriting is allowed) for authentication purposes and submitted on Canvas. Please do NOT include any personal information such as your name and netID in your file. Late homework will NOT be accepted. It is your responsibility to MAKE SURE THAT YOUR SUBMISSIONS ARE SUCCESSFUL AND YOUR FILES ARE LEGIBLE AND COMPLETE. It is also your responsibility that whoever reads your work will understand and enjoy it. Up to 1 point out of 10 may be taken off if your solutions are hard to read or poorly presented.

Workshop 7

1. a. Let $\mathfrak{X}$ be the **standard basis** $\{(1,0,0),(0,1,0),(0,0,1)\}$ of $\mathbb{R}^3$. In Workshop 3 we saw that $\mathfrak{Y} = \{(1,1,0),(0,0,1),(1,0,1)\}$ is another basis. Write down $[id_{\mathbb{R}^3}]_{\mathfrak{Y}\mathfrak{X}}, [id_{\mathbb{R}^3}]_{\mathfrak{X}\mathfrak{Y}}$ (*these are called* ***change of basis matrices***) and compute $[id_{\mathbb{R}^3}]_{\mathfrak{Y}\mathfrak{X}}[id_{\mathbb{R}^3}]_{\mathfrak{X}\mathfrak{Y}}, [id_{\mathbb{R}^3}]_{\mathfrak{X}\mathfrak{Y}}[id_{\mathbb{R}^3}]_{\mathfrak{Y}\mathfrak{X}}$. *In general, change of basis matrices are invertible, and every invertible matrix is a change of basis matrix.*

b. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation with $[T]_{\mathfrak{X}\mathfrak{X}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

Compute $[T]_{\mathfrak{Y}\mathfrak{X}}, [T]_{\mathfrak{X}\mathfrak{Y}}, [T]_{\mathfrak{Y}\mathfrak{Y}}$.

2. Let $S : \mathcal{P}_1 \to \mathcal{P}_2$ be the linear transformation given by $S(p(x)) = \int_0^x p(t)dt$ for all $p(x)$ in $\mathcal{P}_1$. Let $\mathfrak{X} = \{1, x\}$ and $\mathfrak{Y} = \{1, x, x^2\}$. Find a basis for $K(S)$ and a basis for $R(S)$.