[redacted]

1a) A = 2×3
B = 3×2

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \ (2\times2)$$

$$BA = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \ (3\times3)$$

Both BA & AB are defined, but their products do not have the same size.

1b) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3+2 & 4+4 \\ 9+4 & 12+8 \end{bmatrix} = \begin{bmatrix} 5 & 8 \\ 13 & 20 \end{bmatrix}$$

$$BA = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3+12 & 6+16 \\ 1+6 & 2+8 \end{bmatrix} = \begin{bmatrix} 15 & 22 \\ 7 & 10 \end{bmatrix}$$

Both AB & BA are defined & have the same size but their products aren't equal. $AB \neq BA$.

2) $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$\begin{aligned} ae + bg &= 0 \\ af + bh &= 0 \\ ce + dg &= 0 \\ cf + dh &= 0 \end{aligned} \quad \Rightarrow \quad \begin{aligned} b &= d \\ e &= f \\ a &= c \\ g &= h \end{aligned} \quad \begin{aligned} ae &= -bg \\ af &= -bh \\ ce &= -dg \\ cf &= -dh \end{aligned}$$

$$\boxed{\begin{bmatrix} -2 & 2 \\ -2 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 2 \\ -2 & -2 \end{bmatrix}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Two 2×2 nonzero matrices that multiply to the 0 matrix

3) Let matrices B and C both be inverses of matrix A.

So: $A^{-1} = B$ and $A^{-1} = C$.

By definition. $AB = BA = I$ and $AC = CA = I$.

$B = BI$
$\quad = B(AC)$
$\quad = (BA)C$
$\quad = IC$
$B = C$

$B = C$, therefore the inverse of A is unique.

4) Suppose A, B are nxn invertible matrices. (Show $(AB)^{-1}$ exists.)

Show AB is invertible:

$(A)(A^{-1}) = (A^{-1})(A) = I$ and $B(B^{-1}) = (B^{-1})(B) = I$.

If AB is invertible, then there exists C st. $(AB)C = C(AB) = I$.

Let $C = B^{-1}A^{-1}$.

$AB(B^{-1}A^{-1}) = A(BB^{-1})A^{-1} = (AI)A^{-1} = AA^{-1} = I$

$(B^{-1}(A^{-1}))(AB) = B^{-1}(A^{-1}A)B = (B^{-1}I)B = B^{-1}B = I.$

Thus $B^{-1}A^{-1}$ is the inverse of AB, and AB is invertible.