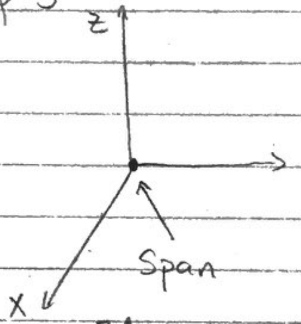
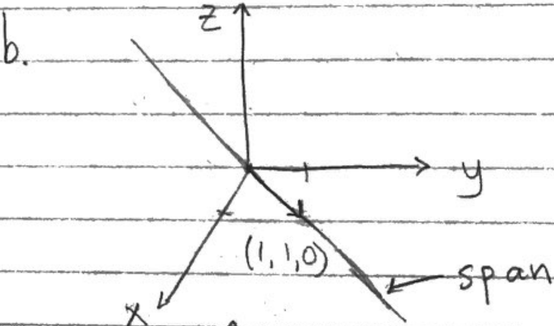
Workshop 3

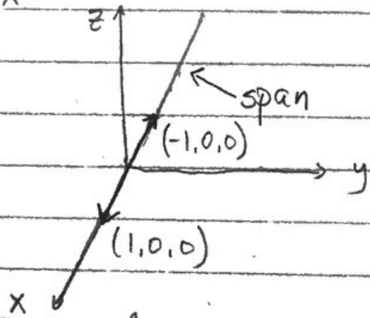
1a.



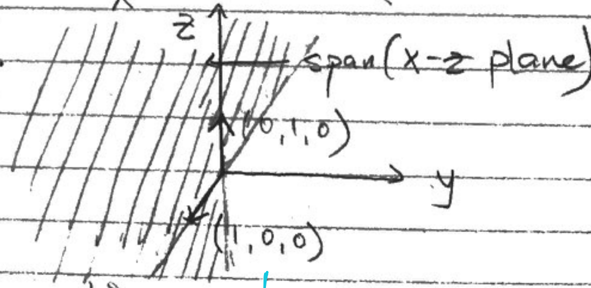
b.



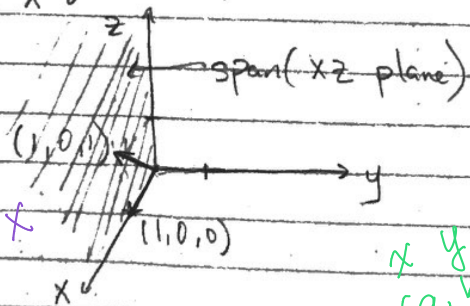
c.

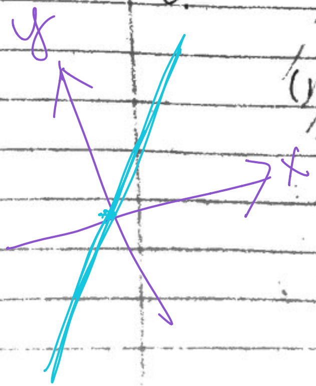


d.



e.



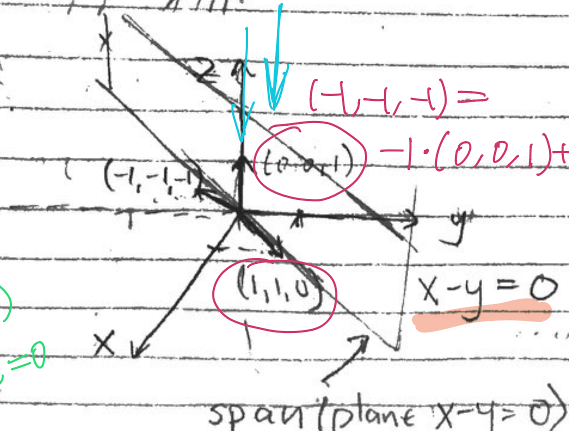


x y z

(a, b, b)

y - z = 0

f.

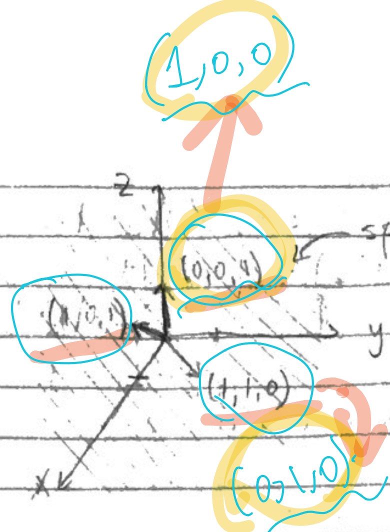


$(-1,-1,-1) = -1\cdot(0,0,1) + (-1)\cdot(1,1,0)$

$a\cdot(0,0,1) + b\cdot(1,1,0) = (b,b,a)$

$a\cdot(-1,-1,-1) + b\cdot(0,0,1) + c\cdot(1,1,0) = (-a+c, -a+c, -a+b)$

g.



$\{(1,0,1), (0,0,1), (1,1,0)\}$

$(a,0,a) + (0,0,b) + (c,c,0)$

$a\cdot(1,0,1)+b\cdot(0,0,1)+c\cdot(1,1,0)$

$= (\underbrace{a+c}_{d},\ \underbrace{c}_{e},\ \underbrace{a+b}_{f})$

$\{(1,0,0),(0,1,0),(0,0,1)\}$

2a. The span of $\{0\}$ is the zero vector because all linear combinations of the zero vector result in the zero vector; $c_1\cdot[0]=0$

b. The span of $\{1+x\}$ is $c_1\cdot[1+x]$ for all values of c_1 because the span is all of the linear combinations of of vector $\{1+x\}$

c. The span of $\{1,-1\}$ is $c_3\cdot[1]$ for all values of c_1. The linear combinations of $\{1,-1\}$ are $c_1\cdot[1]+c_2\cdot[-1]$, which can be simplified to $(c_1-c_2)\cdot[1]$. Since (c_1-c_2) is an arbitrary constant, the span can be represented as $c_3\cdot[1]$ for all multiples of c_3

d. The span of $\{1,x^2\}$ is $c_1[1]+c_2[x^2]$. The linear combinations ~~of~~ of $\{1,x^2\}$ are $c_1\cdot[1]+c_2[x^2]$, so that is the span of this set of vectors.

e. The span of $\{1,1+x^2\}$ is $c_3[1]+c_2[x^2]$ for all values of c_1 and c_2. The linear combinations of $\{1,1+x^2\}$ are $c_1[1]+c_2[1+x^2]$, which can be simplied to $c_1+c_2+c_2x^2$ and rewritten as $(c_1+c_2)+c_2x^2$. Since (c_1+c_2) is an arbitrary constant, the span is equavalent to $c_3+c_2x^2$, where c_3, c_2 can be any value

f. The span of $\{1+x, x^2, -1-x-x^2\}$ is $c_4[1+x]+c_5[x^2]$. The linear combinations of $\{1+x, x^2, -1-x-x^2\}$ can be written as $c_1[1+x]+c_2[x^2]+c_3[-1-x-x^2]$. This can be further simplified to $(c_1-c_3)+(c_1-c_3)x+(c_2-c_3)x^2$. Since (c_1-c_3) and (c_2-c_3) are arbitrary constants, the span can be rewritten as $c_4[1]+c_4[x]+c_5[x^2]$

9. The span of $\{1+x, x^2, 1+x^2\}$ is P_2. The linear combinations of this set of vectors is $c_1[1+x]+c_2[x^2]+c_3[1+x^2]$, which can be simplified to $(c_1+c_3)[1]+c_1[x]+(c_2+c_3)[x^2]$. Since the coefficients are arbitrary, this can be rewritten to $c_4+c_5x+c_6x^2$, and this is the same form as P_2

3a.

From Q1

$$c_1\begin{bmatrix}0\\0\\0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix} \longrightarrow \begin{bmatrix}0\\0\\0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

a) Vector $\{(0,0,0)\}$ is linearly dependent

$$c_1\begin{bmatrix}1\\1\\0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix} \qquad c_1=0$$

b) Vector $\{(1,1,0)\}$ is linearly independent

$$c_1\begin{bmatrix}1\\0\\0\end{bmatrix}+c_2\begin{bmatrix}-1\\0\\0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$c_1-c_2=0$

$c_1=c_2$

c) Vectors $\{(1,0,0),(-1,0,0)\}$ are linearly dependent

$$c_1\begin{bmatrix}1\\0\\0\end{bmatrix}+c_2\begin{bmatrix}0\\0\\1\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$c_1=0 \qquad c_2=0$

d) Vectors $\{(1,0,0),(0,0,1)\}$ are linearly independent

$$c_1\begin{bmatrix}1\\0\\0\end{bmatrix}+c_2\begin{bmatrix}1\\0\\1\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$c_1+c_2=0 \qquad c_1=0$

$c_2=0$

e) Vectors $\{(1,0,0),(1,0,1)\}$ are linearly independent

$$c_1\begin{bmatrix}1\\1\\0\end{bmatrix}+c_2\begin{bmatrix}0\\0\\1\end{bmatrix}+c_3\begin{bmatrix}-1\\-1\\-1\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$c_1 - c_3 = 0$ $\quad c_1 = c_3$

$c_1 - c_3 = 0$

$c_2 - c_3 = 0$ $\quad c_2 = c_3$

$c_1 = c_2 = c_3$ $\quad \mathbb{R}^3$

f) Vectors $\{(1,1,0),(0,0,1),(-1,-1,-1)\}$ are linearly dependent

$$c_1\begin{bmatrix}1\\1\\0\end{bmatrix}+c_2\begin{bmatrix}0\\0\\1\end{bmatrix}+c_3\begin{bmatrix}1\\0\\1\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$c_1 + c_3 = 0$ $\quad c_3 = 0$ $\quad c_1(1,1,0)+c_2(0,0,1)+c_3(-1,-1,-1) = (0,0,0).$

$c_1 = 0$

$c_2 + c_3 = 0$ $\quad c_2 = 0$

g) Vectors $\{(1,1,0),(0,0,1),(1,0,1)\}$ are linearly independent

b. A set of three vectors is linearly dependent in $\mathbb{R}^3$ if the third vector lies on the span of the first two vectors. For example, if the span of the first two vectors is a plane in space, then the set would be linearly dependent if the third vector was on that plane. If that is case, then the third vector is a linear combination of the first two vectors, so it is redundant. However, if the third vector is not on the plane (eg $\perp$ to the plane), then the set is linearly independent since the third vector adds to the span (the span becomes $\mathbb{R}^3$ instead of the plane).

c.

From Q2

$$c_1\begin{bmatrix}0\\0\\0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$ $\vec{v}$ is a vector of coefficients ($c_1\vec{v}=\vec{0}$)

a) Vector $\{0\}$ is linearly dependent

$$c_1\begin{bmatrix}1\\1\\0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$ $c_1 = 0$

Vector $\{1+x\}$ is linearly independent

$$c_1\begin{bmatrix}1\\0\\0\end{bmatrix} + c_2\begin{bmatrix}-1\\0\\0\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$c_1 - c_2 = 0$

$c_1 = c_2$

Vectors $\{1, -1\}$ are linearly dependent

d.
$$c_1\begin{bmatrix}1\\0\\0\end{bmatrix} + c_2\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$c_1 = 0$

$c_2 = 0$

Vectors $\{1, x^2\}$ are linearly independent

e.
$$c_1\begin{bmatrix}1\\0\\0\end{bmatrix} + c_2\begin{bmatrix}1\\0\\1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$c_1 + c_2 = 0 \qquad c_1 = 0$

$c_2 = 0$

Vector $\{1, 1+x^2\}$ are linearly independent

f.
$$c_1\begin{bmatrix}1\\1\\0\end{bmatrix} + c_2\begin{bmatrix}0\\0\\1\end{bmatrix} + c_3\begin{bmatrix}-1\\-1\\-1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$c_1 - c_3 = 0 \qquad c_1 = c_3$

$c_1 - c_3 = 0$

$c_2 - c_3 = 0 \qquad c_2 = c_3$

$c_1 = c_2 = c_3$

$c_1(1+x) + c_2x^2 + c_3(-1-x-x^2) = 0.$

$(c_1 - c_3) + (c_1 - c_3)x + (c_2 - c_3)x^2 = 0.$

$\underbrace{(c_1 - c_3)}_{=0} + \underbrace{(c_1 - c_3)}_{=0}x + \underbrace{(c_2 - c_3)}_{=0}x^2$

Vectors $\{1+x, x^2, -1-x-x^2\}$ are linearly dependent

g.
$$c_1\begin{bmatrix}1\\1\\0\end{bmatrix} + c_2\begin{bmatrix}0\\0\\1\end{bmatrix} + c_3\begin{bmatrix}1\\0\\1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$c_1 + c_3 = 0 \qquad c_1 = 0 \qquad c_2 + c_3 = 0 \qquad c_3 = 0 \qquad c_2 = 0$

Vectors $\{1+x, x^2, 1+x^2\}$ are linearly independent

4. The results from 1 and 2 are equivalent because the vectors created from the polynomial coefficients are the same as the vectors in 1.