Workshop #1: 5/26

1) $\begin{cases} x+y+z=1 \\ x+y-z=1 \end{cases}$ $\Rightarrow$ Solution S is the set...

$x+y+z = x+y-z$ ... such that $z=0$ and the sum of x and y is 1.

$z=-z$

$\boxed{z=0}$

$x+y+0=1$

$\boxed{x+y=1}$

$\begin{pmatrix} 3-2+0=1 \\ 3-2-0=1 \end{pmatrix}$

$\Rightarrow$ One solutions is $(3,-2,0)$.

Multiply $(\cdot)$: 3 is an element of $\mathbb{R}$.

$3\cdot(3,-2,0) = (9,-6,0)$

$(9,-6,0)$ is not a unique solution to the system of equations b/c $9+(-6)+0 \neq 1$.

Thus $(9,-6,0)$ is not in S.

There is no closure under scalar multiplication. Thus, S is not a vector space over $\mathbb{R}$.

2) a. Let $x, y \in \mathbb{R}$ and $(x,y) \in \mathbb{R}^2$.
Let $a, b \in \mathbb{R}$ and $(a,b) \in \mathbb{R}^2$.

$(x,y) + (a,b) = (a+x, b+y)$

$(a+x) \in \mathbb{R}$ and $b+y \in \mathbb{R}$, thus $(a+x, b+y) \in \mathbb{R}^2$.

Let $d \in \mathbb{R}$.

$d\cdot(x,y) = (dx, dy)$

$dx, dy \in \mathbb{R}$, thus $(dx, dy) \in \mathbb{R}^2$.

For any element (a,b) and (x,y) in $\mathbb{R}^2$, $(a,b)+(x,y)$ is a unique element in $\mathbb{R}^2$.
For any element $d \in \mathbb{R}$ and $(x,y) \in \mathbb{R}^2$, $d\cdot(x,y)$ is a unique element in $\mathbb{R}^2$.

2) a.. continued...

(+) 1) Let $u = (u_1, u_2)$, $v = (v_1, v_2)$. $u, v \in \mathbb{R}^2$.

$u+v = (u_1, u_2) + (v_1, v_2) = (u_1+v_1, u_2+v_2)$
$v+u = (v_1, v_2) + (u_1, u_2) = (u_1+v_1, u_2+v_2)$ $\Big\}$ $u+v = v+u$.

2) Let $w = (w_1, w_2)$, $w \in \mathbb{R}$.

$(u+v) + w = ((u_1, u_2) + (v_1, v_2)) + (w_1, w_2) = (u_1+v_1+w_1, u_2+v_2+w_2)$

$u + (v+w) = (u_1, u_2) + ((v_1, v_2) + (w_1, w_2)) = (u_1+v_1+w_1, u_2+v_2+w_2)$

$(u+v) + w = u + (v+w)$.

3) Let the zero element in $\mathbb{R}^2$ be $(0,0)$.

$0+v = (0,0) + (v_1, v_2) = (0+v_1, 0+v_2) = (v_1, v_2) = v$.

Thus, $0+v = v$ for all $v \in \mathbb{R}^2$.

4) Let $-v = (-v_1, -v_2) \in \mathbb{R}^2$.

$v + (-v) = (v_1, v_2) + (-v_1, -v_2) = (v_1 - v_1, v_2 - v_2) = (0,0) = 0$.

Thus $v + (-v) = 0$.

$(\cdot)$ 1) Let $1 \in \mathbb{R}$.
$1 \cdot v = 1 \cdot (v_1, v_2) = (v_1 \cdot 1, v_2 \cdot 1) = (v_1, v_2) = v \in \mathbb{R}^2$

2) Let $a, b, \in \mathbb{R}$.
$ab \cdot v = ab \cdot (v_1, v_2) = (abv_1, abv_2)$
$a \cdot (b \cdot v) = a \cdot (b \cdot (v_1, v_2)) = a \cdot (bv_1, bv_2) = (abv_1, abv_2)$

Thus $ab \cdot v = a \cdot (b \cdot v)$

3) $a \cdot (u+v) = a \cdot ((u_1, u_2) + (v_1, v_2)) = a \cdot (u_1+v_1, u_2+v_2) = (au_1+av_1, au_2+av_2)$
$a \cdot u + a \cdot v = a \cdot (u_1, u_2) + a \cdot (v_1, v_2) = (au_1, au_2) + (av_1, av_2)$
$= (au_1 + av_1, au_2 + av_2)$.

Thus $a \cdot (u+v) = a \cdot u + a \cdot v$.

4) $(a+b) \cdot v = (a+b) \cdot (v_1, v_2) = ((a+b)v_1, (a+b)v_2) = (av_1 + bv_1, av_2 + bv_2)$
$a \cdot v + b \cdot v = a \cdot (v_1, v_2) + b \cdot (v_1, v_2) = (av_1, av_2) + (bv_1, bv_2)$
$= (av_1 + bv_1, av_2 + bv_2)$.

Thus $(a+b) \cdot v = a \cdot v + b \cdot v$. $\boxed{\text{Thus } \mathbb{R}^2 \text{ is a vector space over } \mathbb{R}.}$