Instructions

1. The statements in Italics are for introducing results and notations that may be used again in this course. You are only required to read and think about them.

2. To receive full credit you must explain how you got your answer.

3. While I encourage collaboration, you must write solutions IN YOUR OWN WORDS. DO NOT SHARE COMPLETE SOLUTIONS before they are due. YOU WILL RECEIVE NO CREDIT if you are found to have copied from whatever source or let others copy your solutions.

4. Workshops must be handwritten (electronic handwriting is allowed) for authentication purposes and submitted on Canvas. Please do NOT include any personal information such as your name and netID in your file. Late homework will NOT be accepted. It is your responsibility to MAKE SURE THAT YOUR SUBMISSIONS ARE SUCCESSFUL AND YOUR FILES ARE LEGIBLE AND COMPLETE. It is also your responsibility that whoever reads your work will understand and enjoy it. Up to 1 point out of 10 may be taken off if your solutions are hard to read or poorly presented.

Workshop 15

1. Let $\{\mathbf{v_1}, ..., \mathbf{v_k}\}$ be an orthonormal basis for a subspace V of $\mathbb{R}^n$. Explain why it makes intuitive sense to have $\mathbf{u} = (\mathbf{u} \cdot \mathbf{v_1})\mathbf{v_1} + ... + (\mathbf{u} \cdot \mathbf{v}_k)\mathbf{v}_k$ for every vector $\mathbf{u}$ in V. Then prove it.

2. Use the Gram-Schmidt process to find an orthogonal basis for

$$\text{Span}\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ -3 \\ -3 \end{bmatrix}, \begin{bmatrix} -2 \\ -2 \\ 2 \\ -2 \end{bmatrix} \right\}.$$

3. Let $\mathfrak{X} = \{\mathbf{u_1}, \mathbf{u_2}\}$, $\mathfrak{Y} = \{\mathbf{v_1}, \mathbf{v_2}\}$ be orthonormal bases for a $2-$dimensional subspace V of $\mathbb{R}^n$.

a. Let $Q = [id_V]_{\mathfrak{X}\mathfrak{Y}}$. Express each entry of Q as a dot product.

b. Show that the columns of Q form an orthonormal set of vectors, and so are the rows.

c. Show that b is equivalent to $Q^T Q = QQ^T = I_2$, i.e., $Q^{-1} = Q^T$.

d. Show that $(Q\mathbf{u}) \cdot (Q\mathbf{v}) = \mathbf{u} \cdot \mathbf{v}$ for any column vectors $\mathbf{u}, \mathbf{v}$ in $\mathbb{R}^2$. (*We say Q preserves dot products.*)