3) continued

New matrix:

$$\begin{bmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{2} & 1 \\ \frac{1}{3} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 \end{bmatrix}$$

Old matrix:

$$\begin{bmatrix} 0 & 0 & 1 & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & 0 & 0 \end{bmatrix}$$

When subtracting $1(I)$ from both, the following systems of equations are found:

New system:

$$-1x_1 + \frac{1}{2}x_3 + \frac{1}{2}x_4 = 0$$

$$\frac{1}{3}x_1 - 1x_2 = 0$$

$$\frac{1}{3}x_1 + \frac{1}{2}x_2 - 1x_3 + \frac{1}{2}x_4 + 1x_5 = 0$$

$$\frac{1}{3}x_1 + \frac{1}{2}x_2 - 1x_4 = 0$$

$$\frac{1}{2}x_3 - x_5 = 0$$

$$-1x_1 + x_3 + \frac{1}{2}x_4$$

$$\frac{1}{3}x_1 \cancel{x} - x_2 = 0$$

$$\frac{1}{3}x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_4 \cancel{x_1} - x_3 = 0$$

$$\frac{1}{3}x_1 + \frac{1}{2}x_2 - x_4 = 0$$

$$x_2 = \frac{1}{3}x_1$$

$x_3 = \frac{3}{2}x_1$ $\quad x_2 = \frac{1}{3}x_1$ $\quad x_4 = \frac{1}{3}x_1 + \frac{1}{6}x_1$ $\quad x_4 = \frac{1}{3}x_1 + \frac{1}{6}x_1 = \frac{1}{2}x_1$

$-1x_1 + \frac{1}{2}x_3 + \frac{1}{4}x_1 = 0 \leftarrow$ $\quad x_4 = \frac{3}{6} = \frac{1}{2}x_1$ $\quad x_3 = x_1 - \frac{1}{4}x_1 \rightarrow x_3 = \frac{3}{4}x_1$

3) continued again

4

By comparing the systems of equations, it is visible that the change made Page 3 rank higher than Page 1, since Page 3 used to be equal to 3/4(Page 1), but now, Page 3 is equal to $\frac{3}{2}$(Page 1).

2) Let A be a square matrix. Since transposition does not change the determinant, $\det(A - \lambda I) = \det((A - \lambda I)^T)$. Since $I^T = I$, $\det(A - \lambda I) = \det(A^T - \lambda I)$. Therefore, A and A^T will have the same characteristic polynomials and eigenvalues.

4.)

non-zero

Let $v_1, v_2, \ldots v_k$ be a set of orthogonal vectors
Let $c_1, c_2, \ldots, c_k$ be constants so that :

$c_1 v_1 + c_2 v_2 + \ldots + c_k v_k = 0$ ⇒ Checking for linear independence

Computing the dot product of v_i and the above linear transformatio gives us

$$(c_1 v_1 + c_2 v_2 + \ldots + c_k v_k) = 0$$
$$v_i \cdot (c_1 v_1 + c_2 v_2 + \ldots + c_k v_k) = 0 \cdot v_i$$
$$c_1 v_i \cdot v_1 + c_2 v_i \cdot v_2 + \ldots + c_k v_i \cdot v_k = 0$$

If $i \neq j$, then $v_i \cdot v_j = 0$ since the vectors are orthogonal. This gives

$$c_i \cdot v_i \cdot v_i = c_i \|v_i\|^2 = 0$$

Since v_i is a non-zero vector, $\|v_i\|$ is also non-zero
This means $c_i = 0$

∴ We can conclude that $c_1 = c_2 = \cdots = c_k = 0$. Hence, a set of non-zero orthogonal vectors is <u>linearly independent</u>