

INSTRUCTIONS

1. The statements in *Italics* are for introducing results and notations that may be used again in this course. You are only required to read and think about them.
2. To receive full credit you must explain how you got your answer.
3. While I encourage collaboration, you must write solutions **IN YOUR OWN WORDS. DO NOT SHARE COMPLETE SOLUTIONS** before they are due. **YOU WILL RECEIVE NO CREDIT** if you are found to have copied from whatever source or let others copy your solutions.
4. Workshops must be handwritten (electronic handwriting is allowed) for authentication purposes and submitted on Canvas. Please do **NOT** include any personal information such as your name and netID in your file. Late homework will **NOT** be accepted. It is your responsibility to **MAKE SURE THAT YOUR SUBMISSIONS ARE SUCCESSFUL AND YOUR FILES ARE LEGIBLE AND COMPLETE**. It is also your responsibility that whoever reads your work will understand and enjoy it. Up to 1 point out of 10 may be taken off if your solutions are hard to read or poorly presented.

WORKSHOP 14

1. Let A be a square matrix such that the entries in each row sum to 1. Show that 1 is an eigenvalue of A .
2. Let A be a square matrix. Show that A and A^T have the same eigenvalues.
3. In the last example discussed during the lecture, suppose the people who own page 3 are infuriated by the fact that it's ranked lower than page 1. In an attempt to boost page 3's ranking, they create a page 5 that links to page 3, and page 3 also links to page 5. Does this boost page 3's ranking above that of page 1? (Hint: You can answer this question with little computation; certainly without getting an eigenvector for the new matrix.)
4. Prove: an orthogonal set of nonzero vectors is linearly independent.
5. Let $\{\mathbf{v}_1, \dots, \mathbf{v}_k\}$ be an orthonormal basis for a subspace V of \mathbb{R}^n , show that $\mathbf{u} = (\mathbf{u} \cdot \mathbf{v}_1)\mathbf{v}_1 + \dots + (\mathbf{u} \cdot \mathbf{v}_k)\mathbf{v}_k$ for every vector \mathbf{u} in V . Explain how this equality makes intuitive sense.
6. Use the Gram-Schmidt process to find an orthogonal basis for

$$\text{Span} \left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ -3 \\ -3 \end{bmatrix}, \begin{bmatrix} -2 \\ -2 \\ 2 \\ -2 \end{bmatrix} \right\}.$$