Workshop 12

1. $\det(A)=0 \rightarrow A\vec{x}=\vec{b},\ \vec{b}\neq 0$

When:

$\det(A)=0$, there can be two types of solutions to $A\vec{x}=\vec{b}$. One is that there can be infinitely many solutions, or no solutions. When $\det(A)=0$, it means that the matrix A is not invertible, which also means that the column vectors are linearly dependent. Which means they are linear combinations of each other. In $A\vec{x}=b$, if vector b is not in the column space of matrix A, it will have no solutions, since b will not be linear combinations of A. If vector b is in the column space of A, it will have infinitely many solutions since b will be linear combinations of A.

Example:

① $\underbrace{\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}}_{A}\begin{pmatrix}x_1\\ x_2\end{pmatrix}=\begin{pmatrix}1\\ 1\end{pmatrix} \rightarrow \det(A)=0$, infinitely many solutions since b is in the column space of A.

② $\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}=\begin{pmatrix}1\\ 0\end{pmatrix} \rightarrow \det(A)=0$, No solutions.

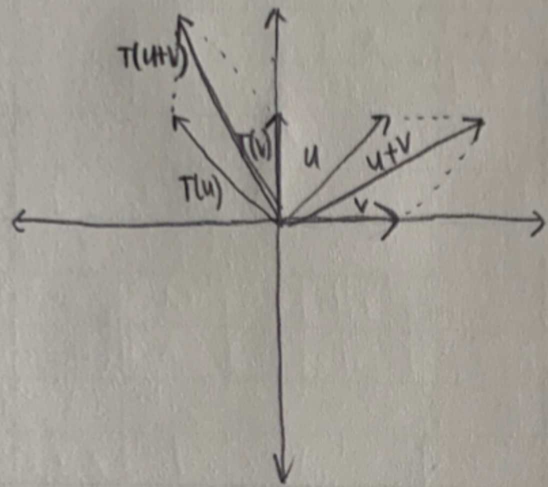
When $\det(A)\neq 0$, there will exist a unique solution of $A\vec{x}=\vec{b}$, when $b\neq 0$. When $\det(A)\neq 0$, it means the matrix A is invertible and we can use its' inverse A^{-1} to show:

$$A\vec{x}=\vec{b}$$
$$(A^{-1}\cdot A)\vec{x}=A^{-1}\vec{b}$$
$$I\cdot\vec{x}=A^{-1}\vec{b}=\vec{x}$$

, which will be a unique solution.

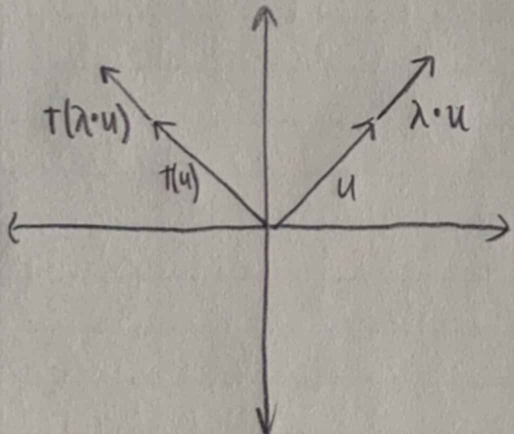
$\rightarrow$

2a)



addition preserved:

$$T(u+v) = T(u) + T(v).$$



scalar multiplication preserved:

$$\lambda \cdot T(u) = T(\lambda \cdot u)$$

2b) T has no eigenvectors or eigen values, since an eigen vector, u, is one such that $T(u) = \lambda u$, and T is a rotational transformation, while multiplying u by λ will only scale u in the same direction, not rotate it.

meaning it changes vector direction