1. Determine whether each of the following statements is true or false. Justify your answer. No credit will be given to answers without correct justification. (8 pts each) * I illustrated using 2x2; can do similarly for 4x4 *

a. Taking the determinant gives a linear transformation from $Mat_{4\times 4}(\mathbb{R})$ to $\mathbb{R}$.

$2 = 2\cdot\det(A)$ #

$\det(2A) = \det\left(\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\right) = 4$

(1) properties of det.

(2) definition of linear transformation.

$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $B = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

$\det: Mat_{4\times 4}(\mathbb{R}) \to \mathbb{R}$

$A \longrightarrow \det(A)$

$\det(A) = 1 = \det(B)$

$A+B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

(2) $\begin{cases} \det(A+B) \stackrel{?}{=} \det(A) + \det(B) & X \\ \det(r\cdot A) \stackrel{?}{=} r\det(A) & X \end{cases}$

$\det(A+B) = 0 \neq$
$\det(A) + \det(B) = 2$

3.

n $A_{n\times n}$

Let **w** be a nonzero column vector (i.e., an $n \times 1$ matrix for some positive integer n). Let $A = \mathbf{w}(\mathbf{w^T})$ (matrix multiplication of **w** with the transpose of **w**). Show that **w** is an eigenvector of A. (10 pts)

$(n\times 1)\times(1\times n) \to n\times n.$

$\vec{w} = \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix}$ $\det(A-\lambda I) = 0$ X

$A\vec{w} = \lambda\vec{w}$?

$A\cdot\vec{w} = \vec{w}\,[(\vec{w}^T)\,\vec{w}]$

$(1\times n)\times(n\times 1) \to 1\times 1$

$\lambda \,/\!/\,$ number $= \vec{w}\cdot\vec{w}$

$\begin{cases} \text{there exist } \lambda\ (= \vec{w}^T\vec{w}) \text{ such} \\ A\vec{w} = \lambda\vec{w} \\ \text{and } \vec{w} \neq \vec{0} \end{cases} \Rightarrow \vec{w}$ is an eigenvector

If $T : \mathbb{R}^{100} \to \mathbb{R}^{400}$ is given by a matrix whose every entry is 1 under some choice of bases, then $K(T)$ (i.e. the kernel of T) has dimension

A

(1) $\dim K(T) + \dim R(T) = 100$

$\dim R(T) =$ dim of column span of $A = 1$

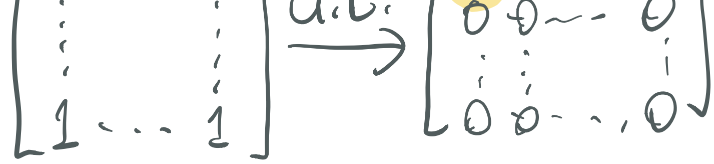
$\Rightarrow \dim K(T) = 99.$

only pivot

99

(2) $A\vec{x} = \vec{0}$ $\begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}$ G.E. $\begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}$

400×100



A is an $m \times 3$ matrix such that the equation $A\mathbf{x} = \mathbf{b}$ has a unique solution for some column vectors $\mathbf{b}$ and no solution for other column vectors $\mathbf{b}$. Fill in each blank below with $=, <$ or $>$.

$\mathbb{R}^3$

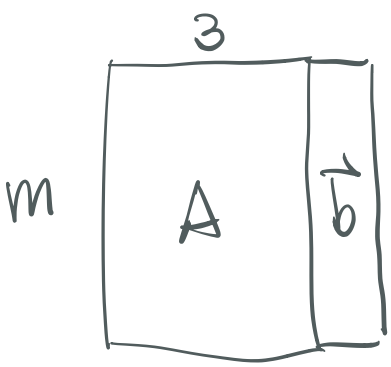
$m > 3$

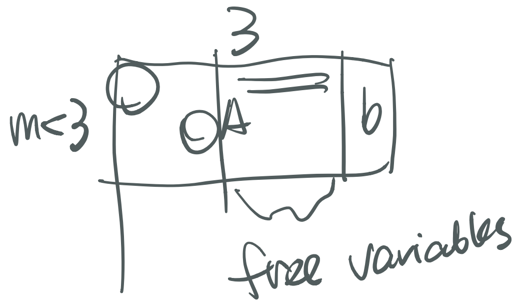
$\text{rank}(A) < 3$

$A\vec{x} = \vec{b}$ has a solution if and only if $\vec{b}$ is in the column span of A.

Not every $\vec{b}$ in $\mathbb{R}^3$ is in the column span of A.

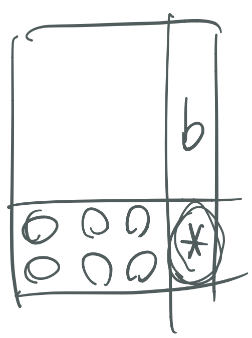
$\Rightarrow \underbrace{\dim(\text{column span of } A)}_{= \ \text{rank}(A)} < 3$





$m = 3$

$m > 3$

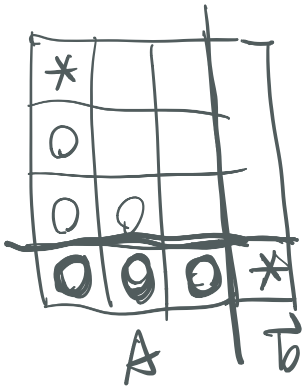


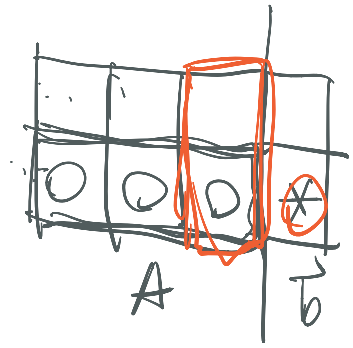
$A: 3\times 3$

$A\vec{x}=\vec{b}$ has unique soln $\Leftrightarrow$ A invertible

A^{-1} A^{-1}

$\underline{\vec{x}}=A^{-1}\vec{b}$





If $m<3$. it's possible $A\vec{x}=\vec{b}$ have no solution

but for other $\vec{b}$ it will have infinite solutions