More Related to Workshop 16 Problem 2b: $[A|b]$

$\left[\begin{array}{cc|c} x & x & x \\ x & x & x \\ 0 & 0 & 1 \end{array}\right]$

$A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$, then $A\vec{x} = \vec{b}$ is inconsistent.

Want to find $\vec{x}$ that minimizes $\|A\vec{x} - \vec{b}\|$.

1st Approach:

Let W = column span of A. $A\vec{x} = \vec{c}$ has a solution exactly when $\vec{c}$ is in W.

Let $\vec{w} = U_W(\vec{b})$, then $A\vec{x} = \vec{w}$ will have a solution $\vec{x}_0$.

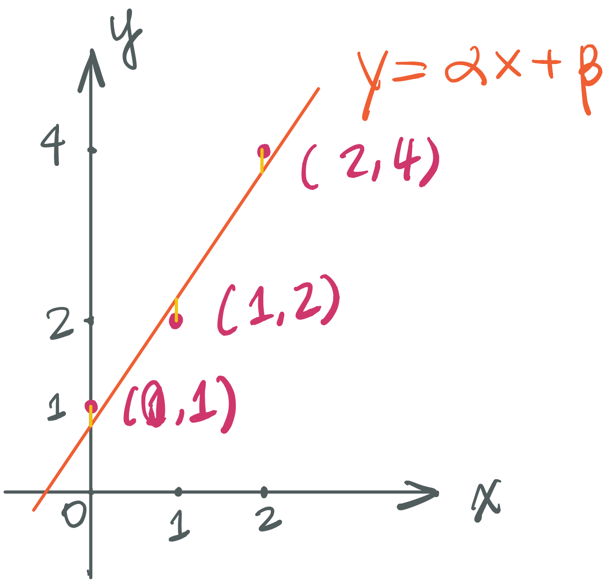
$\|A\vec{x}_0 - \vec{b}\| = \|\vec{w} - \vec{b}\|$ (this is as small as $\|A\vec{x} - \vec{b}\|$ can get!)

$\vec{0} = A^T\vec{z} = A^T(\vec{b} - \vec{w}) = A^T\vec{b} - A^T\vec{w} = A^T\vec{b} - A^TA\vec{x}_0$

So solve $A^TA\vec{x} = A^T\vec{b}$!

2nd Approach:

Use multivariable calculus.



Least squares

$$\begin{cases} \alpha \cdot 0 + \beta = 1 \\ \alpha \cdot 1 + \beta = 2 \\ \alpha \cdot 2 + \beta = 4 \end{cases}$$

$$\underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix}}_{A} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \underbrace{\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}}_{\vec{b}}$$

Def. A matrix A with real entries is called symmetric if

$A^T = A$.

Theorem. Symmetric matrices are diagonalizable over $\mathbb{R}$.
* In particular, all eigenvalues of symmetric matrices are real.

→ Eigenspaces will have dimension = multiplicities of λ

Theorem. If $\vec{u}$ and $\vec{v}$ are eigenvectors of a symmetric matrix A with distinct eigenvalues λ, μ, then $\vec{u} \cdot \vec{v} = 0$

find basis for each eigenspace

Gram-Schm...

Proof:

$$A\vec{u} = \lambda\vec{u}, \quad A\vec{v} = \mu\vec{v}$$

$(AB)^T = B^T A^T$

$$(A\vec{u}) \cdot \vec{v} = \lambda \vec{u} \cdot \vec{v}, \quad \vec{u} \cdot A\vec{v} = \mu \vec{u} \cdot \vec{v}$$

$$\|$$

$$(A\vec{u})^T \vec{v} = \vec{u}^T A^T \vec{v} = \vec{u}^T A \vec{v}$$

$$\Rightarrow \lambda \vec{u} \cdot \vec{v} = \mu \vec{u} \cdot \vec{v}, \text{ i.e., } (\lambda - \mu)\vec{u} \cdot \vec{v} = 0 \overset{\lambda-\mu \neq 0}{\Longrightarrow} \vec{u} \cdot \vec{v} = 0$$

orthonormal eigenbasis! ...

Corollary. Symmetric matrices are orthogonally diagonalizable, i.e., there exist an orthogonal matrix Q and diagonal matrix D such that $A = QDQ^T$.