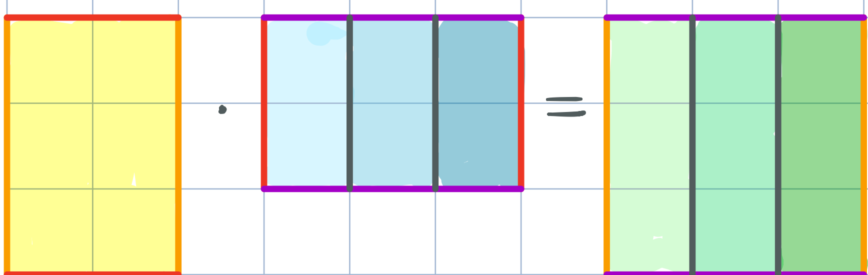
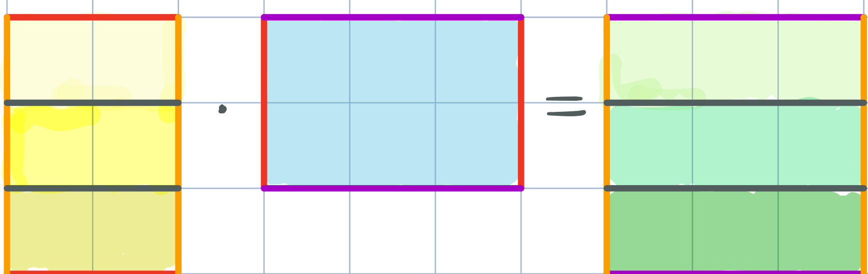
* Since we can multiply 2 nonzero matrices to get the zero matrix, it doesn't make sense to divide by a matrix in general. However, we'd still like to do "divide" when we can. This brings us to the notion of invertibility of a matrix. Here we restrict our attention to square matrices because there multiplication preserves the size.

Def. An $n \times n$ matrix is called invertible if there exists an $n \times n$ matrix B such that $AB = BA = I_n$.





More generally:

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \\ A_{31} & A_{32} \end{bmatrix} \cdot \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \\ C_{31} & C_{32} \end{bmatrix}$$

$$C_{11} = A_{11}B_{11} + A_{12}B_{21} \qquad C_{12} = A_{11}B_{12} + A_{12}B_{22}$$

$$C_{21} = A_{21}B_{11} + A_{22}B_{21} \qquad C_{22} = A_{21}B_{12} + A_{22}B_{22}$$

$$C_{31} = A_{31}B_{11} + A_{32}B_{21} \qquad C_{32} = A_{31}B_{12} + A_{32}B_{22}$$

E.g.

$$\begin{bmatrix} 0 & A \\ I_2 & 0 \end{bmatrix} \cdot \begin{bmatrix} B & C & D \\ E & 0 & F \end{bmatrix} = \begin{bmatrix} AE & 0 & AF \\ B & C & D \end{bmatrix}.$$