Def. A subset W of a vector space V is called a subspace if it contains 0 (the zero element/vector) and is closed under $+$ and $\cdot$

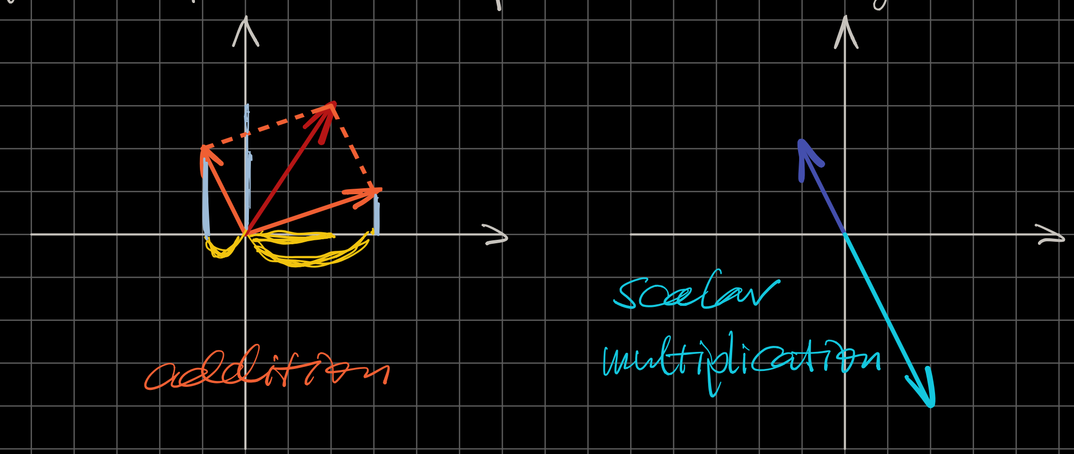
Say V is a vector space over $\mathbb{R}$, then this means:

① for any w_1, w_2 in W, $w_1 + w_2$ is in W (a priori they're just in V)

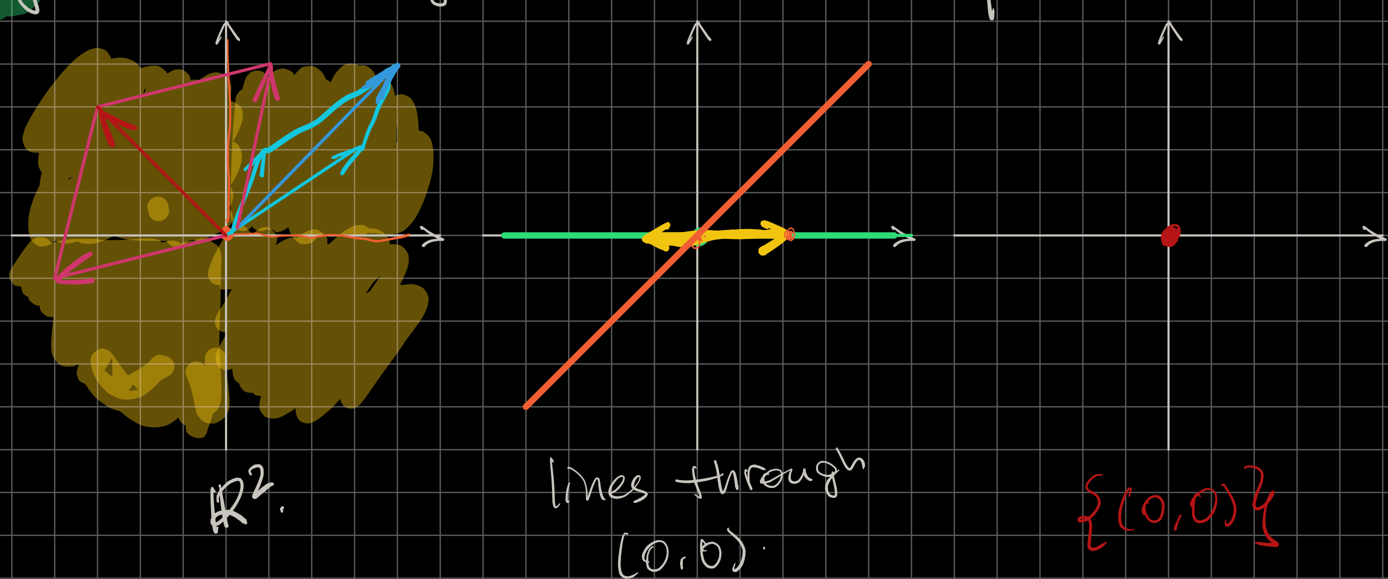
② for any r in $\mathbb{R}$ and w in W, $r \cdot w$ is in W.

(can replace $\mathbb{R}$ with $\mathbb{C}$)

Recall from Workshop 1 Problem 2b:

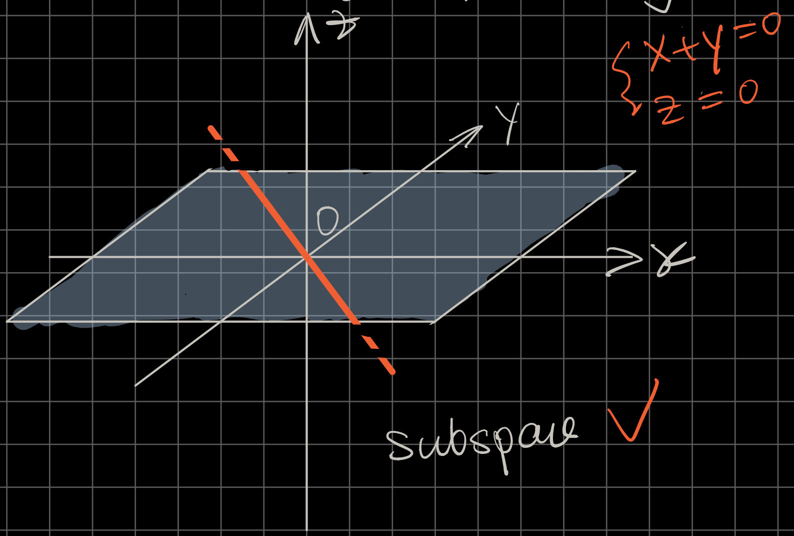


Eg. The following are all the subspaces of $\mathbb{R}^2$:



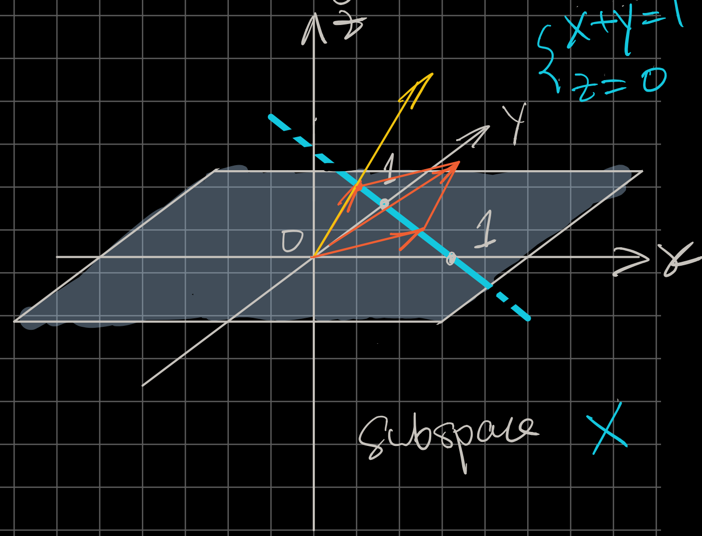
E.g. We can view the set of solutions to $\begin{cases} x+y+z=0 \\ x+y-z=0 \end{cases}$ as a subset of $\mathbb{R}^3$:

$\Updownarrow$

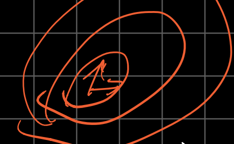


We can also view the set of solutions to $\begin{cases} x+y+z=1 \\ x+y-z=1 \end{cases}$ as a subset of $\mathbb{R}^3$:

$\Updownarrow$



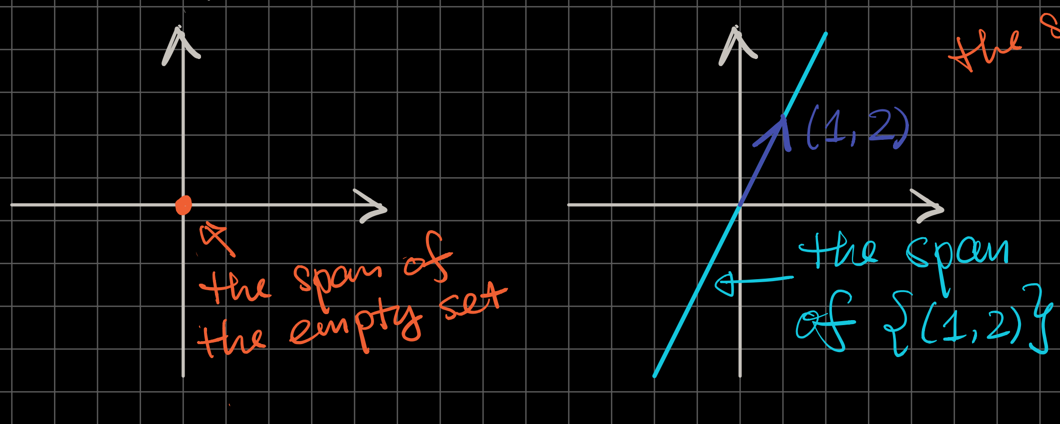
Natural question: Given a set of vectors in a vector space V, what is the smallest subspace containing this set of vectors?

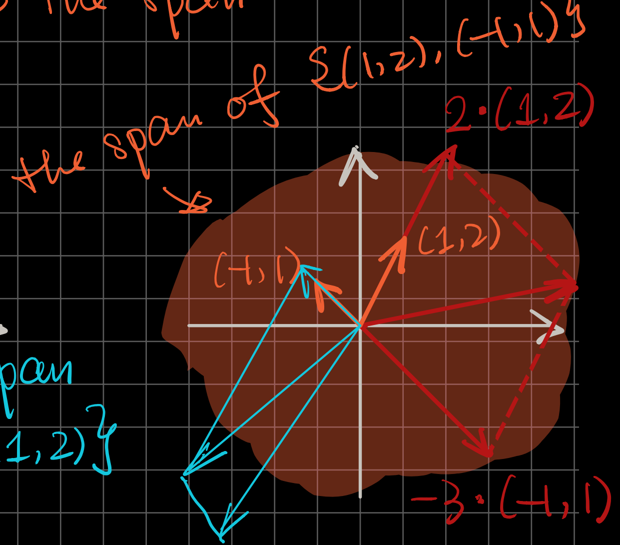


Def. The span of a set S of vectors in a vector space V is the smallest subspace of V that contains S.

every subspace that contains S contains the span

Eg. Span of vectors in $\mathbb{R}^2$





Observation: the span of a nonempty subset S of V consists exactly of all vectors of the form

$a_1 v_1 + \cdots + a_k v_k$, where k is some natural number, $a_1, \cdots, a_k$ are in $\mathbb{R}/\mathbb{C}$, and $v_1, \cdots, v_k$ are in S