Instructions

1. The statements in Italics are for introducing results and notations that may be used again in this course. You are only required to read and think about them.

2. To receive full credit you must explain how you got your answer.

3. While I encourage collaboration, you must write solutions IN YOUR OWN WORDS. DO NOT SHARE COMPLETE SOLUTIONS before they are due. YOU WILL RECEIVE NO CREDIT if you are found to have copied from whatever source or let others copy your solutions.

4. Homework must be handwritten (electronic handwriting is allowed) for authentication purposes and submitted on Canvas. Please do NOT include any personal information such as your name and netID in your file. Late homework will NOT be accepted. It is your responsibility to MAKE SURE THAT YOUR SUBMISSIONS ARE SUCCESSFUL AND YOUR FILES ARE LEGIBLE AND COMPLETE. It is also your responsibility that whoever reads your work will understand and enjoy it. Up to 4 points out of 40 may be taken off if your solutions are hard to read or poorly presented.

Homework 6

1. Find the least squares solution $\boldsymbol{x_0}$ to $A\boldsymbol{x} = \boldsymbol{b}$, where $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix}$, $\boldsymbol{b} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$. Then check that $A\boldsymbol{x_0} = \boldsymbol{w}$ for the $\boldsymbol{w}$ you found in Workshop 16 Problem 2b (6 pts).

2. Suppose that a spring whose natural length is L inches is attached to a wall. A force y is applied to the free end of the spring, stretching the spring s inches beyond its natural length. Hooke's law states (within certain limits) that $y = ks$, where k is a constant called the spring constant. Now suppose that after the force y is applied, the new length of the spring is x. Then $s = x - L$, and Hooke's law yields $y = ks = k(x - L) = a + kx$, where $a = -kL$. Apply the method of least squares to the following data to estimate k and L (8 pts):

Length x in inches	Force y in pounds
3.5	1.0
4.0	2.2
4.5	2.8
5.0	4.3

3. Use the method of least squares to find the parabola that best fits the data (9 pts):

$$(0, 2), (1, 2), (2, 4), (3, 8)$$

4. Determine whether this statement is true or false and explain why: for any inconsistent system of linear equations $A\mathbf{x} = \boldsymbol{b}$, the vector $\boldsymbol{x_0}$ for which $\|A\boldsymbol{x_0} - \boldsymbol{b}\|$ is a minimum is unique. (6 pts)

5. Let $A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$. Find an orthogonal matrix Q and diagonal matrix D such that $A = QDQ^T$. (11 pts)