Instructions

1. The statements in Italics are for introducing results and notations that may be used again in this course. You are only required to read and think about them.

2. To receive full credit you must explain how you got your answer.

3. While I encourage collaboration, you must write solutions IN YOUR OWN WORDS. DO NOT SHARE COMPLETE SOLUTIONS before they are due. YOU WILL RECEIVE NO CREDIT if you are found to have copied from whatever source or let others copy your solutions.

4. Homework must be handwritten (electronic handwriting is allowed) for authentication purposes and submitted on Canvas. Please do NOT include any personal information such as your name and netID in your file. Late homework will NOT be accepted. It is your responsibility to MAKE SURE THAT YOUR SUBMISSIONS ARE SUCCESSFUL AND YOUR FILES ARE LEGIBLE AND COMPLETE. It is also your responsibility that whoever reads your work will understand and enjoy it. Up to 4 points out of 40 may be taken off if your solutions are hard to read or poorly presented.

Homework 5

1. Suppose that $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w} \in \mathbb{R}^m$ are vectors satisfying:

$$\|\boldsymbol{u}\| = 2 \quad \|\boldsymbol{v}\| = 3 \quad \|\boldsymbol{w}\| = 4 \quad \boldsymbol{u} \cdot \boldsymbol{v} = -1 \quad \boldsymbol{u} \cdot \boldsymbol{w} = 2 \quad \boldsymbol{v} \cdot \boldsymbol{w} = -2$$

Compute the following expressions: (2 pts each)

a. $(2\boldsymbol{u} + \boldsymbol{v}) \cdot (3\boldsymbol{v} - 4\boldsymbol{w})$
b. $\|\boldsymbol{u} + \boldsymbol{v}\|^2$
c. $\|-6\boldsymbol{w}\|$
d. $\|2\boldsymbol{v} - \boldsymbol{w}\|$

2. Use dot products to represent $\boldsymbol{u} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$ as a linear combination of the vectors in the orthogonal set $S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} \right\}$. (4 pts)

3. Find an orthogonal matrix with first column $\begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{bmatrix}$. (7 pts)

4. When do we have $||\boldsymbol{u} + \boldsymbol{v}|| = ||\boldsymbol{u}|| + ||\boldsymbol{v}||$ for vectors $\boldsymbol{u}, \boldsymbol{v}$ in $\mathbb{R}^n$? Explain why. (Hint: Think geometrically.) (4 pts)

5. Finish Workshop 16 Problem 2b. (5 pts)

6. Let $\boldsymbol{u} = \begin{bmatrix} 1 \\ 4 \\ -1 \end{bmatrix}$ and $S = \left\{ \frac{1}{\sqrt{6}} \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \right\}$.

a. Check that S is orthonormal. (4 pts)

b. Find the vector $\boldsymbol{w}$ in the span of S that is closest to $\boldsymbol{u}$. (4 pts)

c. Find the distance between $\boldsymbol{w}$ and $\boldsymbol{u}$. (4 pts)