Instructions

1. The statements in Italics are for introducing results and notations that may be used again in this course. You are only required to read and think about them.

2. To receive full credit you must explain how you got your answer.

3. While I encourage collaboration, you must write solutions IN YOUR OWN WORDS. DO NOT SHARE COMPLETE SOLUTIONS before they are due. YOU WILL RECEIVE NO CREDIT if you are found to have copied from whatever source or let others copy your solutions.

4. Homework must be handwritten (electronic handwriting is allowed) for authentication purposes and submitted on Canvas. Please do NOT include any personal information such as your name and netID in your file. Late homework will NOT be accepted. It is your responsibility to MAKE SURE THAT YOUR SUBMISSIONS ARE SUCCESSFUL AND YOUR FILES ARE LEGIBLE AND COMPLETE. It is also your responsibility that whoever reads your work will understand and enjoy it. Up to 4 points out of 40 may be taken off if your solutions are hard to read or poorly presented.

Homework 3

1. Using Gaussian Elimination, either find the general or unique solution of the system, or explain why it is **inconsistent** (i.e., has no solution) (2 pts each):

a. $\begin{cases} -2x_2 + x_3 + 2x_4 = 0 \\ 2x_1 + x_3 + 4x_4 = 1 \end{cases}$

b. $\begin{cases} -2x_2 + x_3 + 2x_4 = 0 \\ 2x_1 + x_3 + 4x_4 = 1 \\ x_1 + x_4 = 2 \\ 2x_1 - x_2 + x_4 = -4 \end{cases}$

c. $\begin{cases} -2x_2 + x_3 + 2x_4 = 0 \\ 2x_1 + x_3 + 4x_4 = 1 \\ x_1 + x_4 = 2 \\ 2x_1 - x_2 + x_4 = -4 \\ 2x_1 - x_2 + x_3 + 3x_4 = -7 \end{cases}$

d. $\begin{cases} -2x_2 + x_3 + 2x_4 = 0 \\ 2x_1 + x_3 + 4x_4 = 1 \\ x_1 + x_4 = 2 \\ 2x_1 - x_2 + x_4 = -4 \\ 2x_1 - x_2 + x_3 + 3x_4 = -7 \\ -3x_1 - 2x_2 + x_3 - x_4 = -5 \end{cases}$

2. $A = \begin{bmatrix} -1 & -2 & -3 & -4 \\ 0 & 2 & 3 & 0 \\ 0 & 2 & 0 & -3 \\ 1 & -2 & 0 & 7 \end{bmatrix}$. Use the matrix inversion algorithm to find the inverse of A or show that A is not invertible (4 pts).

3. Let $T : \mathbb{R}^5 \to \mathbb{R}^4$ be the linear transformation given by the matrix

$$\begin{bmatrix} 0 & 1 & 2 & 1 & 1 \\ 1 & 2 & 4 & 5 & -3 \\ 2 & 5 & 10 & 11 & -7 \\ 1 & 1 & 2 & 4 & -2 \end{bmatrix}$$

under standard bases (i.e., this is the matrix $[T]_{\mathfrak{X}\mathfrak{Y}}$, where

$$\mathfrak{X} = \{(1,0,0,0),(0,1,0,0),(0,0,1,0),(0,0,0,1)\}$$

and

$$\mathfrak{Y} = \{(1,0,0,0,0),(0,1,0,0,0),(0,0,1,0,0),(0,0,0,1,0),(0,0,0,0,1)\}$$

are the standard basis for $\mathbb{R}^4$ and $\mathbb{R}^5$, respectively).

a. Find a basis for $K(T)$ and $R(T)$, respectively (3 pts each).

b. Find the reduced row echelon form of the matrix (1 pt).

c. What is the rank of the matrix (1 pt)?

4. a. Use Gaussian elimination to show that $\mathfrak{X} = \{(0,2,4),(1,2,3),(5,2,0)\}$ and $\mathfrak{Y} = \{(1,2,3),(0,1,4),(5,6,0)\}$ are bases for $\mathbb{R}^3$ (3 pts each).

b. Let $\mathfrak{Z} = \{(1,0,0),(0,1,0),(0,0,1)\}$. Use the matrix inversion algorithm to compute $[id_{\mathbb{R}^3}]_{\mathfrak{X}\mathfrak{Z}}$ and $[id_{\mathbb{R}^3}]_{\mathfrak{Y}\mathfrak{Z}}$ (3 pts each).

c. Compute $[id_{\mathbb{R}^3}]_{\mathfrak{X}\mathfrak{Y}}$ (2 pts).

d. Write each of $[id_{\mathbb{R}^3}]_{\mathfrak{Z}\mathfrak{X}}$ and $[id_{\mathbb{R}^3}]_{\mathfrak{X}\mathfrak{Z}}$ as a product of elementary matrices (3 pts each).