Instructions

1. The statements in Italics are for introducing results and notations that may be used again in this course. You are only required to read and think about them.

2. To receive full credit you must explain how you got your answer.

3. While I encourage collaboration, you must write solutions IN YOUR OWN WORDS. DO NOT SHARE COMPLETE SOLUTIONS before they are due. YOU WILL RECEIVE NO CREDIT if you are found to have copied from whatever source or let others copy your solutions.

4. Homework must be handwritten (electronic handwriting is allowed) for authentication purposes and submitted on Canvas. Please do NOT include any personal information such as your name and netID in your file. Late homework will NOT be accepted. It is your responsibility to MAKE SURE THAT YOUR SUBMISSIONS ARE SUCCESSFUL AND YOUR FILES ARE LEGIBLE AND COMPLETE. It is also your responsibility that whoever reads your work will understand and enjoy it. Up to 4 points out of 40 may be taken off if your solutions are hard to read or poorly presented.

Homework 2

1. Compute the following products of partitioned matrices using block multiplication:

a. $\begin{bmatrix} 1 & 3 & 1 \end{bmatrix} \left[\begin{array}{c|c} 1 & 2 \\ -1 & 1 \\ 0 & 1 \end{array}\right]$. (2 pts)

b. $\left[\begin{array}{c|c|c} 1 & -1 & 0 \\ 0 & 1 & 2 \end{array}\right] \left[\begin{array}{c} 1 \\ \hline 3 \\ \hline 2 \end{array}\right]$. (2 pts)

c. $\left[\begin{array}{cc} 3 & 0 \\ 0 & 3 \\ \hline 2 & 0 \\ 0 & 2 \end{array}\right] \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. (2 pts)

2. Compute $\begin{bmatrix} I_n & O \\ B & C \end{bmatrix} \begin{bmatrix} A & B^{-1} \\ O & I_n \end{bmatrix}$, where A, B, C are $n \times n$ matrices, O is the $n \times n$ **zero matrix** (i.e. a matrix whose entries are all 0), and I_n is the $n \times n$ identity matrix. (4 pts)

3. Let $S : \mathcal{P}_1 \to \mathcal{P}_2$ be the linear transformation given by $S(p(x)) = \int_0^x p(t)dt$ for all $p(x)$ in $\mathcal{P}_1$. Let $\mathfrak{X} = \{1, x\}$ and $\mathfrak{Y} = \{1, x, x^2\}$. Find a basis for $K(S)$ and a basis for $R(S)$. (6 pts)

4. *Verify that $\mathbb{C}$, together with the ususal addition and scalar multiplication, is a vector space over $\mathbb{R}$.*

a. Show that $\mathfrak{X} = \{1, i\}$ and $\mathfrak{Y} = \{1, i+1\}$ are bases for $\mathbb{C}$. (8 pts)

b. Show that the map $T : \mathbb{C} \to \mathbb{C}$ given by complex conjugation (i.e., $T(a+bi) = a - bi$ for a, b in $\mathbb{R}$) is a linear transformation. (4 pts)

c. Compute $[id_{\mathbb{C}}]_{\mathfrak{X}\mathfrak{Y}}$, $[id_{\mathbb{C}}]_{\mathfrak{Y}\mathfrak{X}}$, $[T]_{\mathfrak{X}\mathfrak{X}}$, $[T]_{\mathfrak{X}\mathfrak{Y}}$, $[T]_{\mathfrak{Y}\mathfrak{X}}$, $[T]_{\mathfrak{Y}\mathfrak{Y}}$. (12 pts)