Instructions

1. The statements in Italics are for introducing results and notations that may be used again in this course. You are only required to read and think about them.

2. To receive full credit you must explain how you got your answer.

3. While I encourage collaboration, you must write solutions IN YOUR OWN WORDS. DO NOT SHARE COMPLETE SOLUTIONS before they are due. YOU WILL RECEIVE NO CREDIT if you are found to have copied from whatever source or let others copy your solutions.

4. Homework must be handwritten (electronic handwriting is allowed) for authentication purposes and submitted on Canvas. Please do NOT include any personal information such as your name and netID in your file. Late homework will NOT be accepted. It is your responsibility to MAKE SURE THAT YOUR SUBMISSIONS ARE SUCCESSFUL AND YOUR FILES ARE LEGIBLE AND COMPLETE. It is also your responsibility that whoever reads your work will understand and enjoy it. Up to 4 points out of 40 may be taken off if your solutions are hard to read or poorly presented.

Homework 1

These short videos will give you the matrix background you need for some of the problems: Intro to Matrices, Operations with Matrices, How to Multpiply Matrices.

1. a.What sized matrices can be multiplied to $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ on the left and right, respectively? Find ALL allowed sizes. (2 pts)

b. What do you get when you do such multiplications? Computing a few examples might help you draw the general conclusion. (2 pts)

c. In general, the **square matrices** (*i.e. matrices with the same number of rows and columns*) with 1's on the diagonal (*when we speak of the diagonal in this class, we always mean the upper-left to lower-right diagonal*) and 0's elsewhere are called **Identity matrices**. Can you generalize your conclusion in b to all identity matrices? Give the general statement. (2 pts)

2. Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \end{bmatrix}$, $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}$, and $C = \begin{bmatrix} c_{11} \\ c_{21} \end{bmatrix}$, where the a_{**}, b_{**}, c_{**} are real numbers. Compute $(AB)C$ and $A(BC)$ and show that they are equal. *One can generalize this and show that for general matrices A, B, and C, $(AB)C = A(BC)$ whenever the products are defined, i.e., matrix multiplication is* ***associative****. For this reason, we can use notations like A^3 and so on without causing confusion.* (4 pts)

3. a. Show that the set of 2×3 matrices with entries in $\mathbb{R}$, together with the usual addition and scalar multiplication, is a vector space over $\mathbb{R}$. *We denote this vector space by $Mat_{2\times 3}(\mathbb{R})$.* (6 pts)

b. Describe the span of the following sets of vectors in the simplest possible terms:

i. $\{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\}$. (2 pts)

ii. $\left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\}$. (2 pts)

iii. $\left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\}$. (2 pts)

c. In b, which sets of vectors are linearly dependent and which sets are linearly independent? (6 pts)

d. Show that matrices with the additional constraint that the two entries on the 3rd column sum to 0 form a subspace of $Mat_{2\times 3}(\mathbb{R})$. (6 pts)

4. By a solution to the differential equation $2y'' + xy' - e^x y = 0$ we mean a twice-differentiable real-valued function $y(x)$ such that the equality $2y'' + xy' - e^x y = 0$ holds for any x for which $y(x)$ is defined. Show that the set of solutions to this differential equation, together with the usual addition and scalar multiplication, form a vector space over $\mathbb{R}$. *A basis for this vector space is called a fundamental set of solutions to this differential equation.* (6 pts)