Consider the following sets of vectors in $\mathbb{R}^2$:

a. $\{(1,2),(3,4),(5,6)\}$

b. $\{(1,1),(-2,-2)\}$

c. $\{(1,-2),(-2,1)\}$

d. $\{(0,0)\}$

Type in all correct answers to each of the following questions.
If there is no correct answer, type in "None".
No justification is needed.

1. Which sets are linearly dependent?

2. The span of which sets have dimension 1?

3. Which sets are bases for $\mathbb{R}^2$?

recall: $\{v_1, \dots, v_n\}$ are l. dependent if exists $a_1 v_1 + \cdots + a_n v_n = 0$ but $a_1, \dots, a_n$ are all 0.

Consider the following sets of vectors in $\mathbb{R}^2$:

basis

a. $\{(0,0)\}$

$\emptyset$ · empty set.

$b \cdot (0,0) = (0,0)$

b. $\{(1,1),(-2,-2)\}$

$\{(1,1)\}$

$(-2,-2) = 2(1,1)$

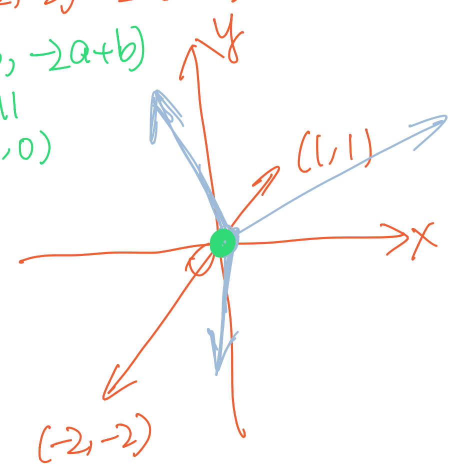
$1 \cdot (-2,-2) - 2 \cdot (1,1) = (0,0)$.

c. $\{(1,-2),(-2,1)\}$

$a(1,-2) + b(-2,1) = (a-2b, -2a+b) = (0,0)$

d. $\{(1,2),(3,4),(5,6)\}$

$\{(1,0),(0,1)\}$



Type in all correct answers to each of the following questions.
If there is no correct answer, type in "None".
No justification is needed.

1. Which sets are linearly dependent? a, b, d.

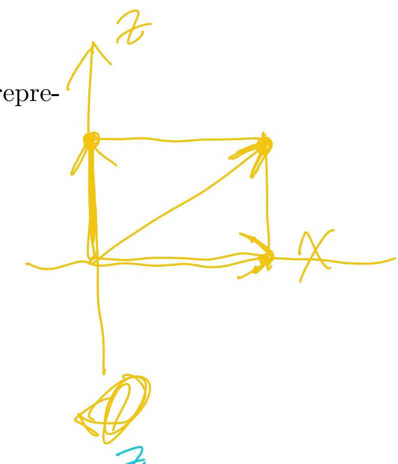
2. The span of which sets have dimension 1? b.

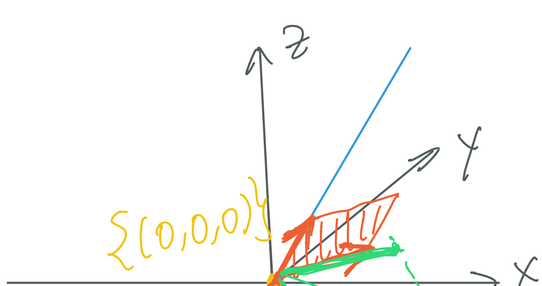
= size of a basis

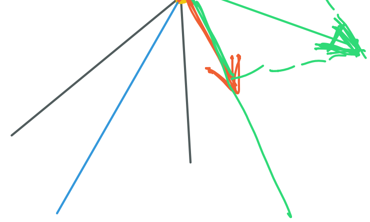
3. Which sets are bases for $\mathbb{R}^2$?

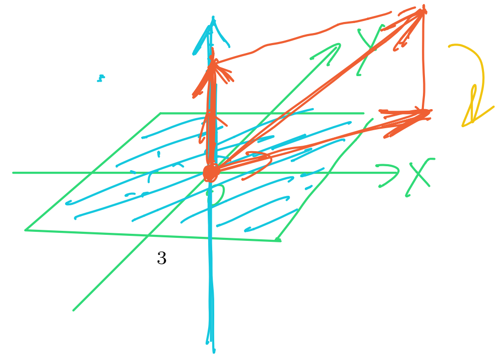
C

Which of the following regions in the $xyz-$coordinate system geometrically represents a subspace of $\mathbb{R}^3$? Select all correct answers.









X The union of the $xy-$plane and the $z-$axis.

√ The $y-$axis.

The plane given by $z = 1$. Doesn't contain $(0,0,0)$.

The region given by $z \geq 0$. $\underbrace{-1\cdot(0,0,1)}_{z\geq 0} = \underbrace{(0,0,-1)}_{z<0}$

Which of the following is a vector space over $\mathbb{R}$?
(Addition and scalar multiplication, if defined, would be the usual ones.)
Select all correct answers.

e.g. $\mathbb{C}$. 1 dim. vector space over $\mathbb{C}$
2 dim. vector space over $\mathbb{R}$. $\{1, i\}$
$Mat_{2\times2}(\mathbb{R})$ is a vector space over $\mathbb{R}$

√ A vector space over $\mathbb{C}$.

the set of all 2x2 matrices / $\mathbb{R}$

The set of invertible 2×2 matrices with entries in $\mathbb{R}$. X

$\begin{pmatrix}0&0\\0&0\end{pmatrix}A \neq I$ $\quad \begin{pmatrix}1&0\\0&1\end{pmatrix} + \begin{pmatrix}-1&0\\0&-1\end{pmatrix} = \begin{pmatrix}0&0\\0&0\end{pmatrix}$

√ The set of real-valued functions with a zero at 2.

f. $f(2)=0$ $\quad g(2)=0$ $\quad (f+g)(2)=f(2)+g(2)=0$ $\quad 0\cdot\begin{pmatrix}1&0\\0&1\end{pmatrix}=\begin{pmatrix}0&0\\0&0\end{pmatrix}$

O The set of real solutions to $x + y = 1$. X

the set of all real-valued functions form a vector space

$(a\cdot f)(2) = a\cdot f(2) = 0$.

Which of the following is a linear transformation of vector spaces over $\mathbb{R}$? Select all correct answers.

$f(x+y) \overset{?}{\neq} f(x)+f(y)$?
$(x+y)+1 \qquad (x+1)+(y+1) = x+y+2$

X The function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x + 1$ for all x in $\mathbb{R}$.

$0 \to 0$?

√ Let V be the vector space of functions from $\mathbb{R}$ to $\mathbb{R}$. The map $T : V \to \mathbb{R}$ given by $\boxed{T(f) = f(1)}$ for each function f in V.

$T(f+g) \overset{?}{=} T(f)+T(g)$
$(f+g)(1) = f(1)+g(1)$

$T(a\cdot f) \overset{?}{=} a\cdot T(f)$
$(a\cdot f)(1) = a\cdot f(1)$

√ Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$. The map $T : Mat_{2\times2}(\mathbb{R}) \to Mat_{3\times2}(\mathbb{R})$ given by $T(X) = AX$ for each X in $Mat_{2\times2}(\mathbb{R})$.

$\underset{2\times2}{X} \xmapsto{T} \boxed{\underset{3\times2\ \ 2\times2}{A\cdot X}}\ 3\times2$

$T(X+Y) \overset{?}{=} T(X)+T(Y)$
$A(X+Y) = AX + AY$

√ The map from $Mat_{2\times2}(\mathbb{R})$ to $\mathbb{R}$ that takes a matrix to the sum of its diagonal

$T(a\cdot X) \overset{?}{=} a\cdot T(X)$
$A\cdot aX = a\cdot AX$

$\begin{pmatrix}a&b\\c&d\end{pmatrix} \xmapsto{T} a+d$

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} a' & b' \\ c' & d' \end{bmatrix}\right) \overset{?}{=} T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) + T\left(\begin{bmatrix} a' & b' \\ c' & d' \end{bmatrix}\right).$$

$$= T\left(\begin{bmatrix} a+a' & b+b' \\ c+c' & d+d' \end{bmatrix}\right) \qquad = (a+d)+(a'+d')$$

4

entries.

$$= (a+a')+(d+d')$$

$$T\left(r\cdot\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) \overset{?}{=} r\cdot T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right)$$

Which of the following is a defined block multiplication that results in a partitioned matrix of the form $\left[\begin{array}{c|c} * & * \\ * & * \\ \hline * & * \end{array}\right]$, where each $*$ represents a number. Select all correct answers.

✓ $\left[\begin{array}{ccc} * & * & * \\ * & * & * \\ \hline * & * & * \end{array}\right] \left[\begin{array}{c|c} * & * \\ * & * \\ * & * \end{array}\right]$

✗ $\left[\begin{array}{cc|c} * & * & * \\ * & * & * \\ * & * & * \end{array}\right] \left[\begin{array}{cc} * & * \\ * & * \\ \hline * & * \end{array}\right]$

✓ $\left[\begin{array}{cc} * & * \\ * & * \\ \hline * & * \end{array}\right] \left[\begin{array}{c|c} * & * \\ * & * \end{array}\right]$

✗ $\left[\begin{array}{c|c} * & * \\ * & * \\ * & * \end{array}\right] \left[\begin{array}{cc} * & * \\ \hline * & * \end{array}\right]$

Recall that $\mathcal{P}_1$ is the vector space of all polynomials of the form $a_0 + a_1 x$, where a_0, a_1 are real numbers. Let $T : \mathcal{P}_1 \to \mathbb{R}$ be the linear transformation given by $T(p(x)) = \int_{-1}^{1} p(x)dx$ for all $p(x)$ in $\mathcal{P}_1$.

a. What are the kernel and range of T? Find a basis for $K(T)$ and a basis for $R(T)$. (6 pts)

b. Let $\mathfrak{X} = \{1, x\}$ and $\mathfrak{Y} = \{1\}$. Compute $[T]_{\mathfrak{Y}\mathfrak{X}}$. (5 pts)

c. Let $\mathfrak{X}' = \{-1, x-1\}$, $\mathfrak{Y}' = \{2\}$, and $\mathfrak{X}, \mathfrak{Y}$ be as in part b. Compute $[T]_{\mathfrak{Y}'\mathfrak{X}}$, $[T]_{\mathfrak{Y}\mathfrak{X}'}$, $\boxed{[T]_{\mathfrak{Y}'\mathfrak{X}'}}$. (9 pts)

a. Kernel is a subspace $\mathcal{P}_1$

range is a subspace of $\mathbb{R}$

$$T(a_0 + a_1 x) = \int_{-1}^{1} (a_0 + a_1 x)\,dx = 2a_0$$

when is this 0?
when $a_0 = 0$!

$K(T) = \{a_1 x\}$ Basis: $\{x\}$

$R(T) = \mathbb{R}$ Basis: $\{1\}$

b.c. It doesn't make sense to write

$\cancel{[T]_{\cancel{x}x}} \cdots$ $T: \overset{\text{dim } 2}{P_1} \to \overset{\text{dim } 1}{\mathbb{R}}$

x, x' y, y'

1×2 $\begin{cases} \boxed{[T]_{yx}},\ [T]_{yx'} \\ [T]_{y'x},\ [T]_{y'x'} \end{cases}$

It does make sense to talk about

2×2 $[id_{P_1}]_{\cancel{x}x'} \cdots$ $id_{P_1} : P_1 \to P_1$

x x'

x' x

1×1 $[id_{\mathbb{R}}]_{yy'}$ - -

$$\begin{cases} [T]_{yx'} = [T]_{yx}\,[id_{P_1}]_{xx'} \\ [T]_{y'x} = [id_{\mathbb{R}}]_{y'y}\,[T]_{yx} \\ [T]_{y'x'} = [id_{\mathbb{R}}]_{y'y}\,[T]_{yx}\,[id_{P_1}]_{xx'} \end{cases}$$

eg. compute $[T]_{y'x'}$:

$T(-1) = \int_{-1}^{1}(-1)dx = -2 = (-1)\cdot 2$

$T(x-1) = \int_{-1}^{1}(x-1)dx = -2 = (-1)\cdot 2.$

$\begin{bmatrix} -1 & -1 \end{bmatrix}$